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ANALYTICAL REPRESENTATION OF THE SECULAR VARIATION

by

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by G. Kauttsleben

SUMMARY

The analytical representation of the main part of the magnetic field is expanded by the description of field dependence on time for the period about from 1850 to 1950. The essential component parts of the secular variation are analyzed.

* * *

The potential of the main part of the magnetic field is computed in the general case with the view of analytical representation of the essential component parts of the magnetic field of the Earth at their global description. To that effect it is necessary to introduce for the coefficients of potential's expression a convenient function of time [1].

If we limit ourselves only to the description of the part of the field induced by inner sources, we may assign for the time dependent potential of the main part of the magnetic field the expression

$$V(r, \theta, \lambda, t) = r_0 \sum_{n=1}^{\infty} \sum_{m=0}^n [c_n^m(t) \cos m\lambda + s_n^m(t) \sin m\lambda] P_n^m(\theta) \left(\frac{r_0}{r}\right)^{n+1}. \quad (1)$$

Here r, θ, λ are the standard geographically oriented spherical coordinates; r_0 is the radius of the Earth assumed to be a sphere; $P_n^m(\theta)$ are the adjoint spherical functions, quasinormalized after Schmidt. We use for time functions convenient time difference polynomials relative to the mean epoch t_0 in the form

$$c_n^m(t) = \sum_{v=0}^{\infty} \gamma_{n,v}^m(t - t_0)^v, \quad s_n^m(t) = \sum_{v=0}^{\infty} \sigma_{n,v}^m(t - t_0)^v. \quad (2)$$

* K ANALITICHESKOMU PREDSTAVLENIYU VEKOVOY VARIATSII

[Communication No. 180 of the Geomagnetic Institute of the Academy of Sciences of East German Democratic Republic]

Compiled in Table 1 are the values of the coefficients $\gamma_{n,v}^m$ and $\sigma_{n,v}^m$ for $K=6$ and $\kappa=3$, obtained by the respective processings of 25 calculations of the potential, known to the year 1959 [2]. The reliability of these values should be investigated, but to that effect we should also check the reliability of separate calculations of the potential, which, as is well known, is quite problematic.

The expression (1) with the time functions (2) give us the full description of the spatial distribution and temporal variation of the main part of the Earth's magnetic field in the corresponding time interval. Hence its partial derivative in time $\partial V / \partial t$ provides a complete description of the spatial distribution and temporal course of the secular variation in corresponding time interval. This representation renders possible the full-value analysis of the properties of the main part of the terrestrial field and its secular variation.

The properties of the representation of the time-dependent potential with the aid of spherical functions compel us to consider two peculiar forms of time variation of the main part of the terrestrial field. In the first case, advantage is taken of the property of spherical functions to represent the field of multipoles placed at coordinate origin. At the same time, the temporal variation of the moments and directions of separate multipoles' axes is investigated. Such considerations have been made by Fankelau and Lucke [3]. The results are not dependent on the rotation of the system of coordinates.

In the second case a particularly interesting property of axis' direction is utilized (line linking both poles $\vartheta=0$ and $\vartheta=\pi$). Since in the system of geographical coordinates this direction coincides with the axis of rotation of the Earth, it offers also a physical interest. Then both structural parts in (1), denoted by similar indices n and m , are reduced to a single structural element, the amplitude of which being invariant relative to the rotation of the system of coordinates near the axis of rotation. Such considerations have been made for example, in [4]. The present note completes them.

TABLE 1

n	m	$\gamma_{n,v}^m$				$\alpha_{n,v}^m$			
		0	1	2	3	0	1	2	3
[10 γ]	[10 γ/year]	[10 γ^2/year^2]	[10 γ^3/year^3]	[10 γ/year]	[10 γ/year]	[10 γ^2/year^2]	[10 γ^3/year^3]	[10 γ/year]	[10 γ/year]
1	0	-3135,3	3,316	-4,671	-55,58				
	1	-228,7	0,5120	-7,999	-1,214	591,0	-0,6058	-0,067	11,10
2	0	-54,5	-0,5247	-9,415	-27,07				
	1	287,2	0,1167	0,939	2,696	-85,5	-1,158	-5,552	-12,42
3	0	94,2	1,980	-5,103	-18,24	122,6	-1,346	-10,86	6,976
	1	-145,8	-0,7178	-1,044	3,100	-31,8	-0,6593	-3,632	17,024
4	0	127,0	-0,7750	4,727	18,58	0,8	0,1209	7,650	-2,308
	1	58,4	0,9213	-1,409	-7,264	52,9	-0,9514	-4,488	2,930
5	0	92,6	0,8000	-5,904	-20,24				
	1	61,3	0,0015	5,128	3,327	29,0	0,2547	-7,956	-5,227
6	0	56,0	0,1887	-2,280	-2,008	-13,5	0,1671	-5,728	-6,609
	1	-35,2	-0,2721	1,467	5,646	-21,2	0,1714	3,528	-2,179
7	0	16,1	0,1822	-0,278	5,282	-9,5	0,1563	-2,474	-5,621
	1	-21,4	-0,2931	1,969	2,331				
8	0	27,4	-0,3623	2,807	11,91	-26,5	0,2748	6,036	0,078
	1	23,3	-0,4871	2,896	11,10	-1,1	0,1185	2,123	-1,829
9	0	0,2	-0,1946	-0,516	4,488	-0,1	0,0638	-0,510	-3,678
	1	-5,7	-0,1454	-1,407	1,794	-8,7	-0,1324	-0,097	1,345
10	0	3,5	-0,1335	-0,979	0,428	0,7	-0,0701	2,860	4,197
	1	10,0	0,3176	-1,011	-12,05				
11	0	5,5	-0,0300	0,792	1,166	8,5	0,0420	-3,673	-4,408
	1	-5,2	-0,0926	0,635	6,495	9,1	0,1057	-0,395	-0,046
12	0	-16,4	-0,1832	-0,622	1,838	-5,4	0,0071	0,804	1,599
	1	-2,3	-0,0189	-0,590	1,852	-1,7	-0,0429	-0,216	2,995
13	0	1,5	0,0043	-0,210	0,242	2,5	-0,1173	0,445	0,133
	1	-1,2	-0,0412	-2,155	-1,979	-2,0	-0,0094	0,861	-1,583

Instead of (1) we obtain

$$V(r, \vartheta, \lambda, t) = r_0 \sum_{n=1}^K \sum_{m=0}^n A_n^m(t) \cos m[\lambda + a_n^m(t)] P_n^m(\vartheta) \left(\frac{r_0}{r} \right)^{n+1}, \quad (3)$$

with the amplitude $A_n^m(t)$ and phase $a_n^m(t)$ being linked with the coefficients in (1) by the correlations

$$A_n^m(t) = \pm \sqrt{[c_n^m(t)]^2 + [s_n^m(t)]^2}, \quad (4)$$

$$\operatorname{tg} m a_n^m(t) = -s_n^m(t) / c_n^m(t). \quad (5)$$

By substitution of (2) $A_n^m(t)$ and $a_n^m(t)$ may be also represented in the form of time polynomials.

For the description of the secular variation we obtain from (3) :

$$\begin{aligned} \frac{\partial V(r, \vartheta, \lambda, t)}{\partial t} = r_0 \sum_{n=1}^K \sum_{m=0}^n & \left\{ \dot{A}_n^m(t) \cos m[\lambda + a_n^m(t)] - \right. \\ & \left. - m A_n^m(t) \dot{a}_n^m(t) \sin m[\lambda + a_n^m(t)] \right\} P_n^m(\vartheta) \left(\frac{r_0}{r} \right)^{n+1}. \end{aligned} \quad (6)$$

Here $\dot{A}_n^m(t)$ is the variation with time of the structural element's amplitude; $\dot{\alpha}_n^m(t)$ is the velocity with which the entire structural element shifts westward, that is, the velocity of field's western drift. The conclusion of northward drift of the configuration elements of the main part of the magnetic field cannot be obtained directly with the help of the analytical representation (1).

The quantities $\dot{A}_n^m(t)$ and $\dot{\alpha}_n^m(t)$ can be directly expressed through the coefficients $c_n^m(t)$, $s_n^m(t)$ with the aid of the correlations (4) and (5)

$$\dot{A}_n^m(t) = \frac{c_n^m(t)\dot{c}_n^m(t) + s_n^m(t)\dot{s}_n^m(t)}{([c_n^m(t)]^2 + [s_n^m(t)]^2)^{1/2}}, \quad (7)$$

$$\dot{\alpha}_n^m(t) = \frac{s_n^m(t)\dot{c}_n^m(t) - c_n^m(t)\dot{s}_n^m(t)}{m \{[c_n^m(t)]^2 + [s_n^m(t)]^2\}}. \quad (8)$$

Applying the equations (2), we may hence too derive specific time functions.

A simple method allows to investigate to what extent the amplitude variations and the western drifts of separate structural elements enter into the general secular variation. For this, only the mean values about the entire surface of the sphere have any significance. It is appropriate to apply a mean value of the form

$$\overline{V^2(t)} = \frac{1}{4\pi} \iint_0^{2\pi} \left(\frac{\partial V}{\partial t} \right)^2 \sin \theta d\theta d\lambda, \quad (9)$$

for on account of orthogonality correlations of spherical functions, it takes the specific form

$$\overline{V^2(t)} = r_0^2 \sum_{n=1}^K \sum_{m=0}^n \left\{ \frac{[\dot{A}_n^m(t)]^2}{2n+1} + \frac{m^2 [\dot{A}_n^m(t)]^2 [\dot{\alpha}_n^m(t)]^2}{2n+1} \right\}. \quad (10)$$

The separate addends $[\dot{A}_n^m(t)]^2$ or $[\dot{\alpha}_n^m(t)]^2$ supply the parts of the secular variation searched for.

In order to familiarize ourselves with the described properties of the secular variation it is sufficient to consider the numerical values of $\dot{A}_n^m(t)$ and $\dot{\alpha}_n^m(t)$ and their contribution to $\overline{V^2(t)}$ for the three selected epochs.

Compiled in Table 2 are the corresponding data, obtained on the basis of the values of Table 1 for the epochs $t_0 = 1860, 1900, 1940$ to $n = m = 4$ inclusive. There also are given the weighted mean values of the western drift with the respective contributions of $[\dot{a}_n^m(t)]^2$ are applied as weights for the potential variation.

TABLE 2

t_0		1860,0				1900,0				1940,0			
n	m	\dot{A}_n^m 10 ⁻³ /year	\dot{a}_n^m %/year	$[\dot{A}_n^m]^2$	$[\dot{a}_n^m]^2$	\dot{A}_n^m 10 ⁻³ /year	\dot{a}_n^m %/year	$[\dot{A}_n^m]^2$	$[\dot{a}_n^m]^2$	\dot{A}_n^m 10 ⁻³ /year	\dot{a}_n^m %/year	$[\dot{A}_n^m]^2$	$[\dot{a}_n^m]^2$
1	0	-1,02	—	0,345	—	-3,32	—	3,65	—	-0,27	—	0,024	—
	1	-0,49	0,09	0,081	0,319	-0,75	0,02	0,187	0,022	-0,01	-0,02	0,000	0,014
2	0	1,07	—	0,229	—	0,52	—	0,055	—	2,57	—	1,328	—
	1	0,35	0,26	0,025	0,324	0,44	0,21	0,039	0,232	1,28	0,32	0,326	0,662
	2	0,04	0,28	0,000	0,462	0,14	0,44	0,004	1,143	0,01	0,35	0,000	0,804
3	0	0,48	—	0,033	—	-0,03	—	0,000	—	0,73	—	0,075	—
	1	0,40	0,25	0,022	0,040	0,84	-0,19	0,101	0,035	0,66	0,02	0,063	0,001
	2	-0,30	0,11	0,013	0,049	-0,77	-0,03	0,086	0,002	0,58	-0,13	0,048	0,043
	3	-0,24	0,18	0,008	0,088	0,04	0,32	0,000	0,250	0,33	0,26	0,016	0,210
4	0	0,30	—	0,010	—	0,80	—	0,071	—	-0,64	—	0,046	—
	1	-0,16	-0,56	0,003	0,050	0,11	-0,19	0,001	0,006	0,35	0,59	0,014	0,067
	2	0,09	-0,22	0,001	0,018	0,14	-0,10	0,002	0,005	0,11	0,28	0,001	0,041
	3	0,23	-0,05	0,006	0,001	0,14	0,13	0,002	0,009	-0,20	0,14	0,004	0,011
	4	0,05	-0,39	0,000	0,024	0,08	-0,17	0,001	0,006	0,50	0,07	0,028	0,002
		Average: 0,17/year				0,37/year				0,32°/year			

The following conclusions are derived from Table 2 : the relative contributions of amplitude variation and of western drift of separate structural elements to the time variation of the potential are very variable in time. The contribution of the western drift prevails essentially over that of amplitude variation. The greatest contribution to western drift of the entire field is made by structural elements with $n = 2$; however, contribution by other structural elements can not be neglected. The structural elements do not show a unique western drift; some of them provide eastern drift. In the whole the reliability is insufficient. The error of the mean values of the western drift must attain at least $\pm 0,1^\circ$ per annum, provided we take into account all sources of errors, including the unreliability of the original data.

Let us note in conclusion that the operations, brought out here, depend on the position of the axis of coordinates $\vartheta = 0$ and can not be carried out for any positions of this axis.

***** THE END *****

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